Determination of the altitude range of a 22 GHz radiometer

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In our group there is no common definition for the altitude range in which a retrieved profile can be regarded as reasonable in terms of sensitivity to perturbations in the true atmospheric profile. So far we have been using the area of the averaging kernels (AoA), which we call measurement response, to get an indication in what altitude range the measurement contributes to what extent to the retrieved profile (compared to the apriori profile). However, the AoA does not contain enough information for a complete definition of the altitude range of the retrieval, because it does not consider if the retrieved profile attributes perturbations in the true atmospheric profile to the correct altitude. A typical set of AVK's of MIAWARA-C (integration time = 24h), indicating why the attribution of perturbations to a certain altitude could be a problem, is shown in Figure 1. While in pressure altitudes between 5 and 0.1 hPa the AVK peak at approximately the altitude they are attributed to (from now on called nominal height) this is not true for higher or lower altitudes. Three quantities to define the altitude range of the retrieval are given in [Rodgers(2000)] on page 57: the first is, that the retrieval is sensitive to the true profile over the height range in which the AoA is approximately 1 (this definition is close to what we have been using up to now) and the second is that the peaks of the AVK should be located at the approximately right level (so perturbations in the true profile are attributed to the right altitude). The third criterion he gives is that the retrieval is reasonable in terms of the transfer function in the altitude range where the widths of the AVK are more or less uniform and comparable with or narrower than those of the original weighting functions.

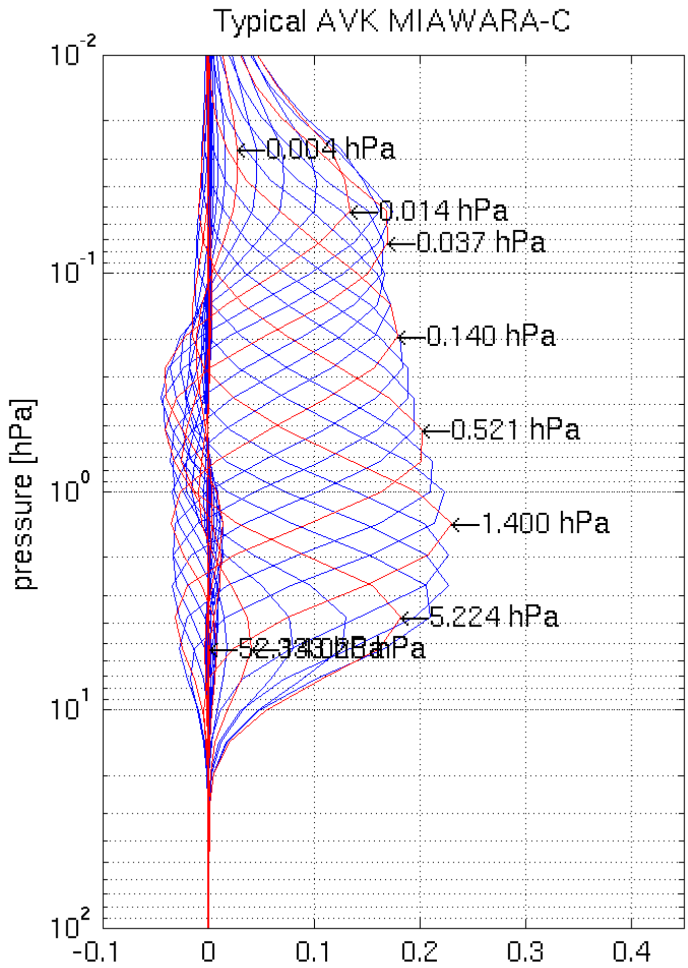

Figure 1: Typical set of AVK

In the first part of this Report the above three quantities are used to determine the altitude range of the retrieved profiles from a typical single day integrated spectrum measured during the Lapbiat campaign (Sodankylä winter 2010). The spectrum was acquired at the 16th of January 2010 and the 24h integration led to a measurement noise of $\sigma = 0.0126$ K which is a typical value for the single day retrievals during the Lapbiat campaign. In the second part of this Report the determined altitude range is applied to the profiles acquired from mid January to mid March 2010 to show that the altitude range determined is reasonable.

1 Averaging Kernels

The averaging kernel matrix, **A**, characterizes the response of the retrieved profile to a perturbation in the true profile. Rodgers interprets this matrix as follows, (p. 47):

> In the case where the state vector represents a profile, the rows a_i^T of **A** can be regarded as smoothing functions: the averaging kernels or model resolution functions. In an ideal inverse method, **A** would be a unit matrix. In reality, rows of **A** are generally peaked functions, peaking at the appropriate level, and with a half width which is a measure of the spatial resolution of the observing system, thus providing a simple characterization of the relationship between the retrieval and the true state. The averaging kernel also has an area, which is found to be approximately unity at levels where the retrieval is accurate, and in general can be thought of as a rough measure of the fraction of the retrieval that comes from the data, rather than the apriori.

In the following the rows of **A**, Rodgers calls them a_i^T, will be called AVK. The averaging kernel matrix is the product of the retrieval gain matrix $\boldsymbol{G_y}$ and the kernel matrix $\boldsymbol{K_x}$:

$$\boldsymbol{A} = \boldsymbol{G_y}\boldsymbol{K_x} \tag{1}$$

$\boldsymbol{K_x}$ has the dimensions $n \times m$, where n is the length of the measurement vector and m the length of the profile vector. The rows of $\boldsymbol{K_x}$ are called weighting functions as they describe the sensitivity of the spectrum to variations in the atmospheric profile. Each element $\boldsymbol{K_x}$ is the partial derivative of a forward model, $\boldsymbol{F_x}$, element with respect to a state vector element, x:

$$K_{ij} = \frac{\partial \boldsymbol{F_i}(x)}{\partial x_j} \tag{2}$$

During the optimal estimation a new state vector is calculated in every iteration starting with the apriori profile. Therefore the weighting function matrix indirectly depends on the apriori profile.
The left-most plot of Figure 2 shows the weighting functions for 20 selected channels of the spectrum of MIAWARA-C. The functions plotted in blue

are at certain frequencies up to 100 MHz away from the line center with the weighting function of the center-most channel reaching the highest up in pressure. They illustrate that the measurements at frequencies far away from the line center do not contribute to the profile at high altitudes and that the further away from the line center the less important is the frequency resolution (e.g. the weighting functions at 50 and 100 MHz away from the line center are similar whereas those 0 and 50 MHz away from the line center are not). The weighting functions plotted in red all belong to channels close to the line center, the lower-most to the one 610 kHz away. Close to the line center there is a significant difference in the weighting functions even for neighbouring channels. Therefore close to the line center a high frequency resolution of the measured spectrum is crucial for the altitude resolution of the retrieved profile.

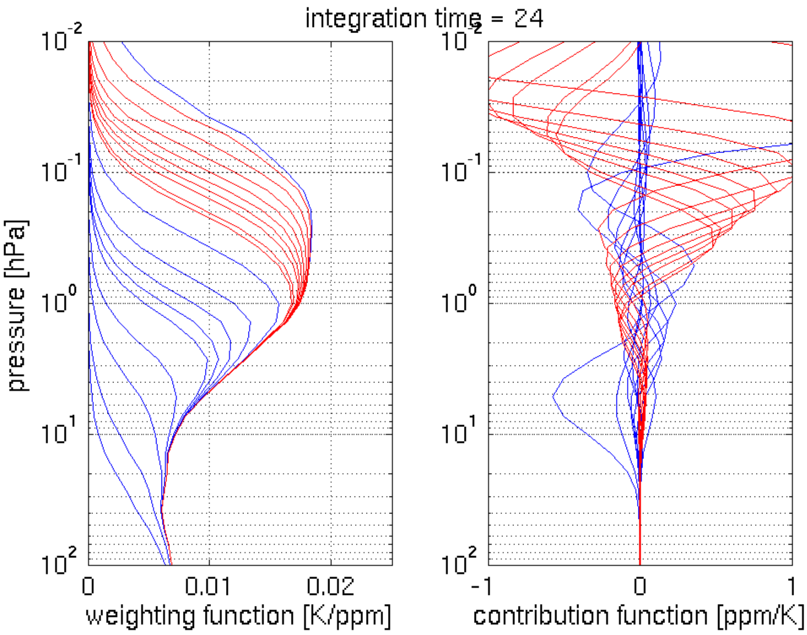

Figure 2: Weighting and contribution functions for 10 channels at 0, 1, 2, 3, 4, 5, 10, 20, 50, 100 MHz from the line center (blue) and 2k channels where k=1:10 from the line center (red).

The retrieval gain matrix $\boldsymbol{G_y}$ has the dimensions $m \times n$ and its columns are called contribution functions as they represent the sensitivity of the retrieved profile to the measurement:

$$\boldsymbol{G_y} = (\boldsymbol{S_a}^{-1} + \boldsymbol{K_x}^T \boldsymbol{S_\epsilon}^{-1} \boldsymbol{K_x})^{-1} \boldsymbol{K_x}^T \boldsymbol{S_\epsilon}^{-1} \tag{3}$$

where $\boldsymbol{S_a}$ is the apriori covariance and $\boldsymbol{S_\epsilon}$ the covariance of the measurement noise (σ). This means the contribution functions directly depend on the measurement noise, the covariance matrix of the apriori profile and indirectly, through the Kernel matrix, on the apriori profile itself.

1.1 Area of the averaging kernels (Measurement Response)

The area of the AVK is approximately unity at levels where the retrieval is sensitive to perturbations in the true profile. In general it can be thought of as a rough measure of the fraction of the retrieval that comes from the measurement, rather than the apriori profile [Rodgers(2000)]. The area of the AVK, *AoA*, for a certain altitude is calculated by simply summing the AVK up:

$$AoA(i) = \Sigma_j AVK(i,j) \tag{4}$$

The AVK and the AoA are shown in Figure 3. In the range where our retrieval is sensitive to perturbations in the true profile the AoA should be close to unity. However, the term 'close to unity' does not give a quantitative definition for the altitude range of the retrieval we are looking for.

As figure 3 shows there are altitudes where the AoA is smaller and altitudes where it is larger than one. The altitudes where the AoA is smaller than one are the altitudes where the apriori profile strongly determines the retrieved profile. It is tempting to interpret this contribution in percent, e.g. an AoA of 0.7 corresponds to an apriori contribution of 30%. I am not sure if such a direct interpretation is correct, because this would mean that AoAs of more than 1 would correspond to negative contributions of the apriori profile. The interpretation of AoA-values larger than one seems to not be straight forward. For MIAWARA-C I use a lower limit of 0.85 to quantify the term 'close to unity' for the AoA. In figure 3 the black vertical line marks the lower limit for the AoA.

The pressure range of our retrieval where the AoA is ≥ 0.85 extends from 4 to 0.02 hPa.

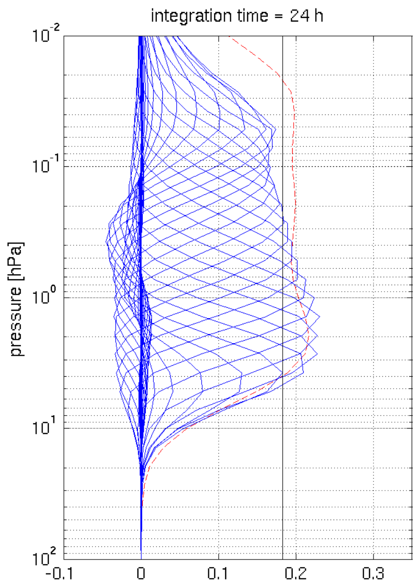

Figure 3: AVK and AoA/5. The black line marks the lower limit for the AoA.

1.2 Peak of the AVK

The peak of the AVK should be located at approximately its nominal height so perturbations in the true profile are attributed to the right altitude. Approximately is a vague definition and my goal is to quantify it. Since we are talking about the correct attribution between the altitude of the retrieved profile compared to the true profile it makes sense to make sure that the nominal height lies well within the altitude range covered by the AVK. Therefore I use 25% of the FWHM as upper limit for the altitude difference between the nominal height of the AVK and its peak.
The peak is found by fitting a polynomial of 3rd degree to values larger than $\frac{2}{3}$ of the estimated peak value of the AVK.
The left-most plot of Figure 4 shows the the peak and the nominal height for selected AVK. The right-most plot of the same Figure displays the difference between the two heights. The black vertical lines mark 25% of the FWHM of the AVK.
According to the result of this method the pressure range of our retrieval setup is from 6 to 0.05 hPa.

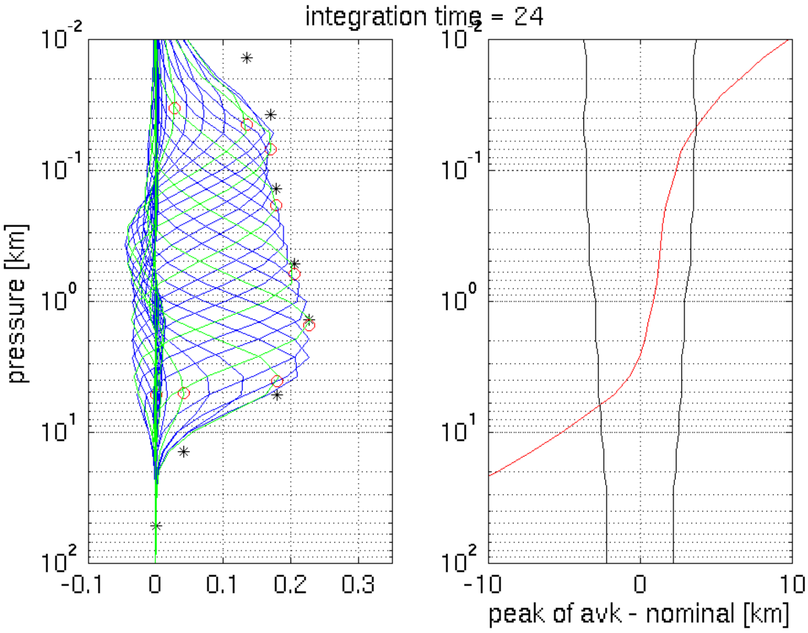

Figure 4: Left: Peak (red circle) and nominal height (black star) for the AVK marked green. Right: Difference between nominal height and height of peak (red) and ±25% of FWHM (black).

1.3 Width of the AVK and the original weighting function

Rodgers mentions that the widths of the AVK should be more or less uniform and comparable or narrower than the ones of the original weighting function at altitudes the retrieval is reasonable from the point of view of the transfer function. Here it is assumed that by weighting functions he means the difference between the weighting functions of the pressure levels the retrieval is calculated on.

To determine this difference weighting functions is not completely straight forward since the weighting functions belong to spectrometer channels and not pressure levels. (I did not find an approach to do the conversion in the literature). The approach used here to convert the weighting functions to difference weighting functions consists of several steps:

1. Take the difference between the weighting function of one channel and the next one.

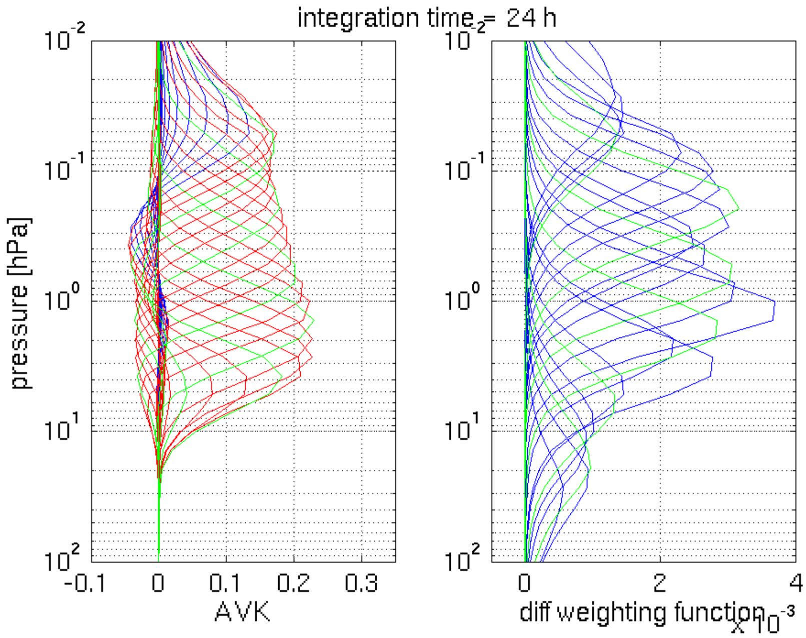

Figure 5: AVK and difference weighting function.

2. Find the pressure at the maximum of each of these first difference weighting functions.
3. Find the difference weighting function with the pressure closest to the middle between two pressure levels of the retrieval.
4. Take the difference between the weighting functions belonging to these difference weighting functions. These are the difference weighting functions searched for.

The instrumental altitude limits for our retrieval are given by the bandwidth (lower limit) and the frequency resolution (upper limit). These limits define the altitude range of the difference weighting functions which in our case is about 40 to 0.02 hPa. Changes in the true profile outside of this altitude range either contribute to the whole spectrum (lower limit) or just to the center channel (upper limit). This means even if some information from outside this altitude range is present in the measured spectrum, it is not possible to attribute it to a certain altitude.
Figure 5 shows the AVK on the left and the difference weighting functions on the right. The AVK marked red are the ones belonging to a nominal pressure a difference weighting function could be calculated. Some AVK and

weighting functions are highlighted in green to fascilitate visible comparison of weighting function and AVK belonging to the same pressure level.

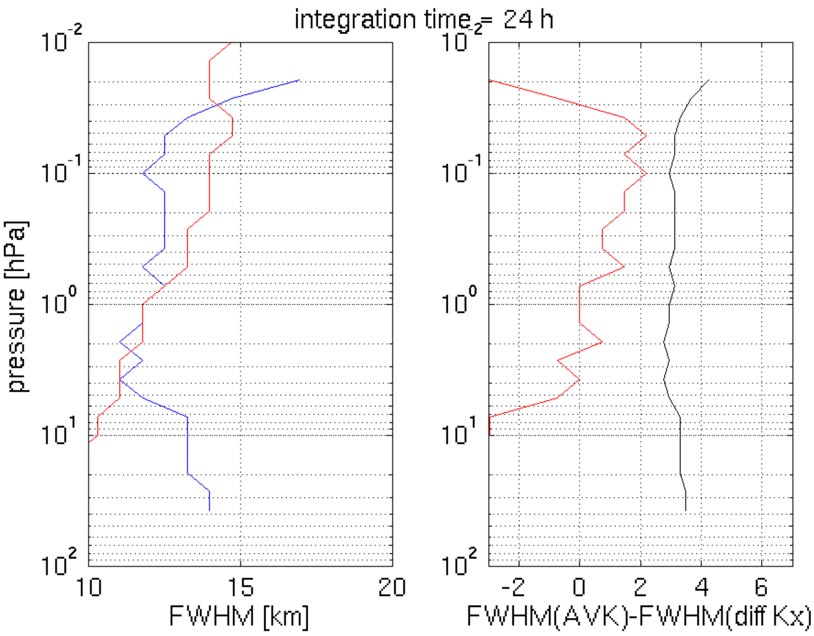

Figure 6: Left: FWHM of difference weighting function (blue) and FWHM of AVK (red). Right: Difference between FWHM of difference weighting function and FWHM of AVK red, 25% of FWHM of difference weighting function.

The left-most plot in Figure 6 shows the FWHM of the weighting function in blue and the one of the AVK in red. The right-most plot shows the difference between the two. The width of our AVK is not completely uniform but linearly increasing towards high altitudes. The same is true for the difference between the FWHM of the AVK and the weighting functions, shown in the plot on the right, as the widths of the latter are more or less uniform. As for the difference between nominal and peak height of the AVK, 25% of the FWHM of the difference weighting function is chosen as upper limit for the difference between the width of the AVK and the weighting functions. The black line in the right-most plot of Figure 6 marks this upper limit. The fact that the difference between the FWHM of the difference weighting function and the one of the AVK is smaller than the upper limit shows that the analyzed retrieval is reasonable in terms of the transfer function at all altitudes a difference weighting fuction can be calculated.
Note: This is not just given for any retrieval. If the measurement noise is too high or if the apriori covariance is too low this criterion is not fulfilled in

the whole altitude range.

1.4 Compilation of the results

In Table 1 a compilation of the altitude ranges determined with the three methods given by Rodgers is displayed. Since all of the three criteria need to be fulfilled at the altitude range a retrieval can be regarded as accurate the highest of the lower limits and the lowest of the upper limits are used. This means during the Lapbiat campaign I consider the single day retrieval of MIAWARA-C as accurate in terms of the defined criteria at a pressure range of 4 - 0.05 hPa.
The upper limit of this altitude range can be increased by longer integration times and a higher frequency resolution of the spectrometer up to the physical limit where the doppler broadening dominates over pressure broadening which is at approximately 0.02 hPa. The lower altitude limit is more difficult to decrease since in addition to the bandwidth of the spectrometer it is basically given by the baseline fit performed during the optimal estimation process. Usually this baseline fit consists of a polynomial of low degree and a linear combination of sine functions removing not only the instrumental baseline but also the information coming from lower layers in the atmosphere from the spectrum.
Outside of the pressure range defined for the retrieval the information content is no zero, but the reliability of the retrieved water vapor vmr decreases in terms of attribution to the right pressure level and independency of the apriori profile.

method	lower limit [hPa]	upper limit [hPa]
AoA	**4**	0.02
peak height	6	**0.05**
width of AVK and weighting functions	40	0.03

Table 1: Pressure limits determined with the three criteria given by Rodgers.

2 The determined altitude range regarded in actual profiles

In this section the altitude limits determined in the previous section are tested on the set of data acquired during the Lapbiat campaign from mid January to mid March. The dataset consists of 59 profiles of which each is compared to the EOS/MLS profile closest in time.
Figure 7 shows the difference between the profiles of MIAWARA-C and the ones of MLS. At altitudes found to be within the range of our retrieval (4-0.05 hPa) the mean difference between the profiles of MIAWARA-C and MLS is within 7%. This indicates that the altitude range determined is reasonable.

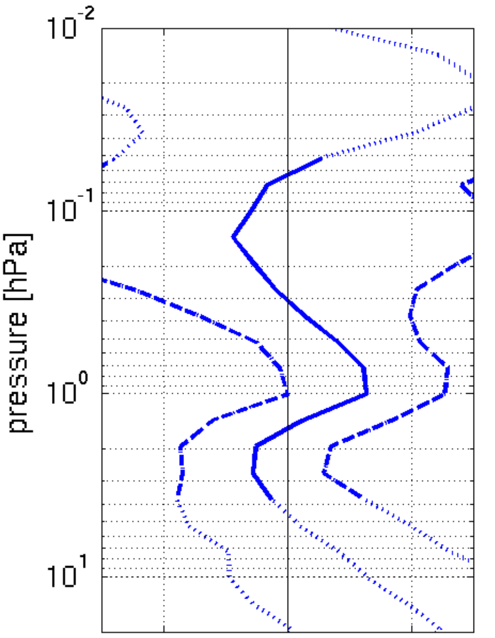

Figure 7: Profiles of MIAWARA-C compared to the profiles of AURA/MLS. The profile in the altitude range of the retrieval is ploted solid.

The profiles of MLS have been interpolated to the pressure grid of MIAWARA-C for this and the following comparison. For comparisons between ground based microwave radiometers and satellites the profile of the satellite is often convolved with the AVK of the microwave instrument to achieve similar altitude resolutions. For this report I decided to not do so because at altitudes where the peak height is far away from the nominal height of the AVK or if the contribution of the apriori profile to the retrieved profile is high, the comparison is distorted by the convolution. The convolution of the satelite profile is achieved by the following formula:

$$\hat{x}_{sat} = x_a + \boldsymbol{A}(x_{sat} - x_a) \tag{5}$$

where x_a is the apriori profile used for the retrieval of MIAWARA-C and x_{sat} the original satellite profile. If the contribution of the measurement in the profile of MIAWARA-C is low the value of the according AVK is low, meaning the convolved satellite profile yields the same vmr as the profile of MIAWARA-C, namely the one of the apriori profile. In short there will be no difference between the profile of MIAWARA-C and MLS at altitudes where the AoA is low, which in my opinion is misleading.
In addition if the peak height of an AVK of MIAWARA-C is far away from its nominal height a certain vmr is attributed to the wrong pressure level. If we now convolve the profile of the satellite with the AVK of MIAWARA-C we introduce the same error to $\hat{x}_{sat}$. Therefore a comparison between the two instruments does not give an indication how well the retrieved vmr values of MIAWARA-C compared to those of MLS at altitudes where there is a big difference between the peak and the nominal height of the AVK.
In Figure 8 time series plots at certain altitude ranges are displayed. The mean values of the measurements in the altitude ranges, indicated in the title of each subplot, is taken in order to account for the different altitude resolution of MIAWARA-C and MLS. The blue crosses show the measurements of MLS and the red dots those of MIAWARA-C. The time series in the altitude range of our retrieval namely the plots for 3 to 1 hPa, 1.0 to 0.3 hPa, 0.3 to 0.1 hPa and 0.10 to 0.03 show a very nice agreement between MIAWARA-C and MLS. For 10 to 3 hPa and 0.03 to 0.01 the time series of the two instruments are still in a reasonable agreement while for altitudes outside these ranges the measurements of MIAWARA-C show almost no time variation which means the contribution of the apriori profile to the measurement is very high.
The comparison of time series of MLS and MIAWARA-C as well as the mean difference profile comparison show that the altitude range determined using the three criteria presented in the first section of this report is reasonable.

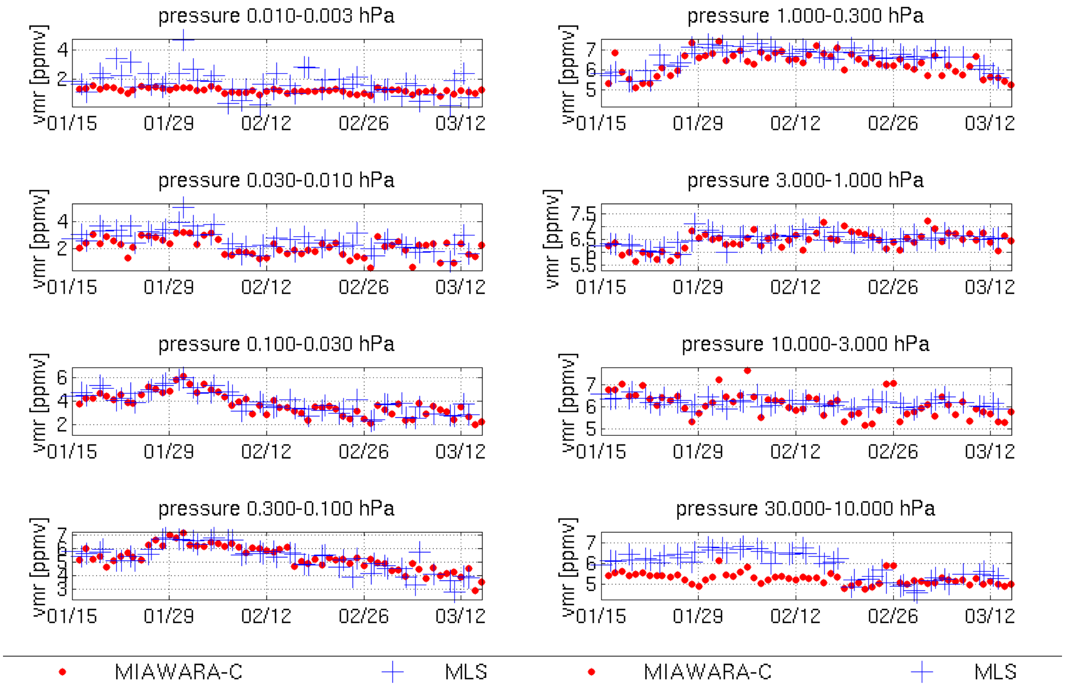

Figure 8: red dots: MIAWARA-C, blue crosses: MLS

3 Conclusions

This report shows a way to define the altitude range of the profile retrieval of our ground based microwave radiometers. The kernel matrix (weighting functions) and the averaging kernel matrix are used to find the range the retrieval of MIAWARA-C during the Lapbiat campaign can be regarded as reliable in terms of certain parameters. The averaging kernel matrix is the product of the kernel matrix and the retrieval gain matrix and it therefor depends on the measurement noise, the apriori covariance matrix and indirectly on the apriori profile.

The AoA (measurement response) indicates the altitude range in which the retrieval is sensitive to perturbations in the true profile, while it is important to also consider the peak height of the AVK to find the altitude range in which those perturbations are attributed to the right altitude. The FWHM of the AVK compared to the one of the difference weighting functions indicate the altitude range in which the retrieval is reasonable in terms of the transfer function.

The altitude ranges determined are valid for the retrieval setup of MIAWARA-C during the Lapbiat campaign. However, the method used for the determination can basically be applied for each retrieval in which averaging kernels are calculated.

References

[Rodgers(2000)] Rodgers, C. D.: Inverse Methodes for Atmospheric Soundings, World Scientific Publishing Co. Pte. Ltd, Singapore, 2000.